Section 1.4 (page 78)

1. (a) 3 (b) 3 (c) 3; $f(x)$ is continuous on $(-\infty, \infty)$.

3. (a) 0 (b) 0 (c) 0; Discontinuity at $x = 3$

5. (a) -3 (b) 3 (c) Limit does not exist.
Discontinuity at $x = 2$

7. $\frac{1}{16}$ **9.** $\frac{1}{10}$

11. Limit does not exist. The function decreases without bound as x approaches -3 from the left.

13. -1 **15.** $-1/x^2$ **17.** $5/2$ **19.** 2

21. Limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

23. 8

25. Limit does not exist. The function approaches 5 from the left side of 3 but approaches 6 from the right side of 3.

27. Discontinuous at $x = -2$ and $x = 2$

29. Discontinuous at every integer

31. Continuous on $[-7, 7]$ **33.** Continuous on $[-1, 4]$

35. Nonremovable discontinuity at $x = 0$

37. Continuous for all real x

39. Nonremovable discontinuities at $x = -2$ and $x = 2$

41. Continuous for all real x

43. Nonremovable discontinuity at $x = 1$
Removable discontinuity at $x = 0$

45. Continuous for all real x

47. Removable discontinuity at $x = -2$
Nonremovable discontinuity at $x = 5$

49. Nonremovable discontinuity at $x = -7$

51. Continuous for all real x

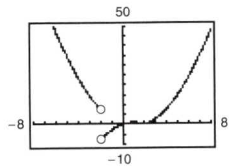
53. Nonremovable discontinuity at $x = 2$

55. Continuous for all real x

57. Nonremovable discontinuities at integer multiples of $\pi/2$

59. Nonremovable discontinuities at each integer

61.



$\lim_{x\to0^+} f(x) = 0$

$\lim_{x\to0^-} f(x) = 0$

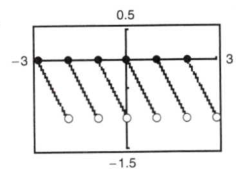
Discontinuity at $x = -2$

63. $a = 7$ **65.** $a = 2$ **67.** $a = -1,\ b = 1$

69. Continuous for all real x

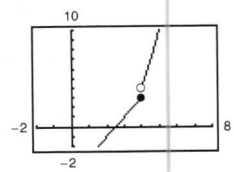
71. Nonremovable discontinuities at $x = 1$ and $x = -1$

73.



Nonremovable discontinuity at each integer

75.

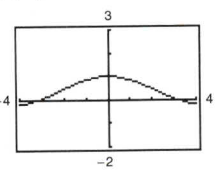


Nonremovable discontinuity at $x = 4$

77. Continuous on $(-\infty, \infty)$

79. Continuous on the open intervals . . . $(-6, -2)$, $(-2, 2)$, $(2, 6)$, ...

81.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

83. Because $f(x)$ is continuous on the interval $[1, 2]$ and $f(1) = 37/12$ and $f(2) = -8/3$, by the Intermediate Value Theorem there exists a real number c in $[1, 2]$ such that $f(c) = 0$.

85. Because $f(x)$ is continuous on the interval $[0, \pi]$ and $f(0) = -3$ and $f(\pi) \approx 8.87$, by the Intermediate Value Theorem there exists a real number c in $[0, \pi]$ such that $f(c) = 0$.

87. 0.68, 0.6823 **89.** 0.56, 0.5636

91. $f(3) = 11$ **93.** $f(2) = 4$

95. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

97. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm1$.

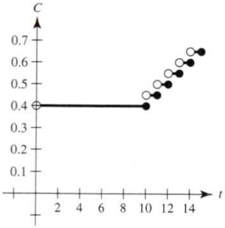
99. True

101. False. A rational function can be written as $P(x)/Q(x)$ where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

103. $\lim_{t \to 4^-} f(t) \approx 28$; $\lim_{t \to 4^+} f(t) \approx 56$

At the end of day 3, the amount of chlorine in the pool is about 28 oz. At the beginning of day 4, the amount of chlorine in the pool is about 56 oz.

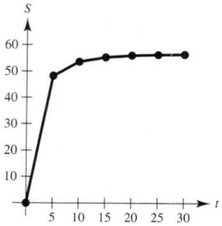
105. $C = \begin{cases} 0.40, & 0 < t \le 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10,\ t \text{ is not an integer} \\ 0.40 + 0.05(t - 10), & t > 10,\ t \text{ is an integer} \end{cases}$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

107–109. Proofs **111.** Answers will vary.

113. (a)

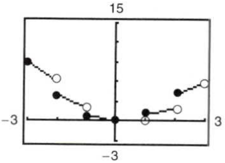


(b) There appears to be a limiting speed, and a possible cause is air resistance.

115. $c = \left(-1 \pm \sqrt{5}\right)/2$

117. Domain: $[-c^2, 0) \cup (0, \infty)$; Let $f(0) = 1/(2c)$

119. $h(x)$ has a nonremovable discontinuity at every integer except 0.



121. Putnam Problem B2, 1988